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EXISTENCE OF PSEUDO ALMOST AUTOMORPHIC MILD SOLUTIONS TO SOME NONAUTONOMOUS SECOND ORDER DIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper we make extensive use of Schauder fixed point principle and exponential stability tools to obtain the existence of pseudo-almost automorphic solutions to some classes of nonautonomous first and second-order abstract differential equations. To illustrate our abstract results, the existence of pseudo almost automorphic solutions to the so called Sine-Gordon boundary value problem will be discussed.

1. Introduction. Fix a Banach space $\mathbf{X}$. This paper is mainly motivated by the paper by Goldstein and N'Guérékata [**21**], in which the existence of *almost automorphic* solutions to the autonomous differential equation

$$\frac{du}{dt} = Au + G(t,u), \quad t \in \mathbf{R} \tag{1.1}$$

where $A : D(A) \subset \mathbf{X} \mapsto \mathbf{X}$ is a closed linear operator on $\mathbf{X}$ which generates an exponentially stable C_0-semigroup $\mathcal{T} = (T(t))_{t\geq 0}$ and the function $G : \mathbf{R} \times \mathbf{X} \mapsto \mathbf{X}$ is given by $G(t,u) = P(t)Q(u)$ with P, Q being continuous functions satisfying some additional conditions, was established. The main tools utilized in [**21**] are fractional powers of linear operators and the well-known Schauder fixed point principle.

This paper has two main goals. The first objective consists of generalizing the result obtained in [**21**] by studying the existence of pseudo almost automorphic solutions to the nonautonomous differential

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