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ON HAAR BASES FOR GENERALIZED DYADIC HARDY SPACES

HUGO AIMAR, ANA BERNARDIS AND LUIS NOWAK

ABSTRACT. In this note we prove that Haar type systems are unconditional bases in the generalized dyadic Hardy space $H_1^{\mathcal{D}}$ in the setting of spaces of homogeneous type. As a consequence, we obtain an alternative proof of the unconditionality of such a basis in Lebesgue spaces on spaces of homogeneous type.

1. Introduction. The theory of Hardy spaces on non-isotropic settings and non-Euclidean spaces is not new. Calderón and Torchinsky [**8**] initiated the study of Hardy spaces on $\mathbf{R}^n$ with anisotropic dilations. Macías [**14**], Coifmann and Weiss [**11**] and Macías and Segovia [**16**] studied Hardy spaces in the general setting of the space of homogeneous type. In 1980 Carleson studied the existence of the unconditional basis on Hardy spaces $H^1(\mathbf{R}^n)$, [**9**]. More precisely, he gives an explicit wavelet basis that is unconditional in $H^1(\mathbf{R}^n)$. In the proof given in [**9**] the regularity of basic functions is crucial. In the books [**13, 20**], conditions on the wavelets for unconditionality in $H^1(\mathbf{R}^n)$ can also be found. These conditions are given in terms of regularity of the functions of the wavelet basis, and therefore the same proof does not hold in the dyadic context for the Haar wavelet in $\mathbf{R}^n$. Notice that, on an abstract metric space (X, d) for which no smoothness better than Lipschitz continuity makes sense, the first basic prototype of localized wavelet is the Haar wavelet. In this note we shall prove that Haar type bases $\mathcal{H}$ are unconditional bases for the atomic dyadic Hardy space $H_1^{\mathcal{D}}$, in the general context of spaces of homogeneous type. As a consequence, we shall obtain a new proof of the unconditionality of Haar systems in Lebesgue spaces on spaces of homogeneous type. We would like to point out that, in [**12**] the authors prove that the usual Haar basis in

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