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BSDES UNDER FILTRATION-CONSISTENT NONLINEAR EXPECTATIONS AND THE CORRESPONDING DECOMPOSITION THEOREM FOR $\mathcal{E}$-SUPERMARTINGALES IN L^p

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ABSTRACT. In this paper, we introduce the notion of $\mathcal{F}_t$-consistent expectation defined on $\mathcal{L}(\Omega, \mathcal{F}, P)$ and prove an existence and uniqueness theorem for solutions and a comparison theorem of BSDE under $\mathcal{E}^\mu$-dominated $\mathcal{F}$-expectations. Furthermore, as an application of this comparison theorem, we obtain the decomposition theorem for $\mathcal{E}$-supermartingales.

1. Introduction. By [**7**], we know that there exists a unique adapted and square integrable solution to a backward stochastic differential equation (BSDE for short) of the type

$$(1) \qquad y_t = \xi + \int_t^T g(s, y_s, z_s)\,\mathrm{d}s - \int_t^T z_s\,\mathrm{d}W_s, \quad 0 \le t \le T,$$

provided the function g is Lipschitz in both variables y and z, and ξ and $(g(t,0,0))_{t\in[0,T]}$ are square integrable. The function g is said to be the generator of BSDE (1). We denote the unique adapted and square integrable solution of BSDE (1) by $(y_t^{(T,g,\xi)}, z_t^{(T,g,\xi)})_{t\in[0,T]}$. When g also satisfies $g(\cdot, y, 0) = 0$ for any $y \in R$, then $y_0^{(T,g,\xi)}$, denoted by $\mathcal{E}_g[\xi]$, is called the g-expectation of ξ; $y_t^{(T,g,\xi)}$, denoted by $\mathcal{E}_g[\xi|\mathcal{F}_t]$, is called the conditional g-expectation of ξ (see [**8**]).

The g-expectation is a kind of nonlinear expectation, which can be considered to be a nonlinear extension of the well-known Girsanov

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