

EXISTENCE-UNIQUENESS OF THE SOLUTION FOR NEUTRAL STOCHASTIC FUNCTIONAL DIFFERENTIAL EQUATIONS

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ABSTRACT. The main aim of this paper is to develop some basic theories of neutral stochastic functional differential equations (NSFDEs). Firstly, we establish a local existence-uniqueness theorem under the local Lipschitz condition for the right side and a lower Lipschitz condition for the left side of the equation. Then continuation theorems and global existence theorems for NSFDEs are obtained. Some classical results, such as the Picard local existence-uniqueness theorem, continuation theorems and the Wintner global existence theorems for deterministic differential equations, are extended to the NSFDEs. Two examples are given to illustrate the efficiency of our results.

1. Introduction. Many physical phenomena can be modeled by stochastic dynamical systems whose evolution on time is governed by random forces as well as intrinsic dependence of the state on a finite part of its past history. Such models may be identified as SFDEs. Neutral stochastic functional differential equations (NSFDEs) not only depend on past and present values but also involve derivatives with delays. In recent years, investigation of NSFDEs has attracted the considerable attention of researchers, and many qualitative properties of solutions to NSFDEs have been obtained [1, 7, 8].

Mao [8] considered the following NSFDE of Itô-type with finite delay

$$(1) \quad d[x(t) - G(x_t)] = f(t, x_t) dt + g(t, x_t) d\omega(t), \quad t_0 \leq t < T,$$

with the initial condition

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