EXISTENCE AND ITERATION OF POSITIVE SOLUTIONS TO THIRD ORDER THREE-POINT BVP WITH INCREASING HOMEOMORPHISM AND POSITIVE HOMOMORPHISM

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ABSTRACT. In this paper, we obtain the existence of positive solutions and establish two corresponding iterative schemes for the following third order three-point boundary value problem

\[
\begin{aligned}
(\phi(u'''))' + q(t) f(u(t)) &= 0, & 0 \leq t \leq 1, \\
u(0) &= \beta u(\xi), & u'(1) = 0, \\
\phi(u''(0)) &= \delta \phi(u''(\xi)),
\end{aligned}
\]

where \(\phi : \mathbb{R} \to \mathbb{R}\) is an increasing homeomorphism and positive homomorphism. The main tool is the monotone iterative technique. An example is given to show our results.

1. Introduction. The purpose of this paper is to consider the existence of positive solutions and establish two corresponding iterative schemes for the following third order three-point boundary value problem (BVP for short),

\[
\begin{aligned}
(\phi(u'''))' + q(t) f(u(t)) &= 0, & 0 \leq t \leq 1, \\
u(0) &= \beta u(\xi), & u'(1) = 0, \\
\phi(u''(0)) &= \delta \phi(u''(\xi)),
\end{aligned}
\]

where \(\phi : \mathcal{R} \to \mathcal{R}\) is an increasing homeomorphism and positive homomorphism with \(\phi(0) = 0\). Here, the following conditions hold:

(H1) \(0 < \xi, \beta, \delta < 1\); 
(H2) \(f : [0, +\infty) \to \mathcal{R}^+\) is continuous and nondecreasing;

Keywords and phrases. Iteration, positive solution, third order three-point boundary value problem, increasing homeomorphism and positive homomorphism.

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