

GENERALIZED M^* -SIMPLE GROUPS

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ABSTRACT. Let X be a compact bordered Klein surface of algebraic genus $p \geq 2$, and let $G = \Gamma^*/\Lambda$ be a group of automorphisms of X where Γ^* is an NEC group and Λ is a bordered surface group. If the order of G is $4q/(q-2)(p-1)$, for $q \geq 3$ a prime number, then the signature of Γ^* is $(0; +; [-]; \{(2, 2, 2, q)\})$. These groups of automorphisms are called generalized M^* -groups. In this paper, we define generalized M^* -simple groups and give some examples of them. Also, we classify solvable generalized M^* -simple groups.

1. Introduction. A compact bordered Klein surface X of algebraic genus $p \geq 2$ admits at most $12(p-1)$ automorphisms [10]. Groups isomorphic to the automorphism group of such a compact bordered Klein surface with this maximal number of automorphisms are called M^* -groups. These groups were first introduced in [11], and have been studied in several papers ([3–5, 9]). Also, the survey article in [5] contains a nice survey of known results about M^* -groups.

An important result about M^* -groups is that they must have a certain partial presentation. This is established by considering an M^* -group as an epimorphic image of a quadrilateral group $\Gamma^*[2, 2, 2, 3]$. A quadrilateral group Γ^* is a non-Euclidean crystallographic (NEC) group with signature

$$(0; +; [-]; \{(2, 2, 2, 3)\}).$$

Also Γ^* is isomorphic to the abstract group with the presentation

$$\langle c_0, c_1, c_2, c_3 \mid c_i^2 = (c_0c_1)^2 = (c_1c_2)^2 = (c_2c_3)^2 = (c_3c_0)^3 = I \rangle.$$

For some bordered surface group Λ the group $G = \Gamma^*/\Lambda$ satisfies $|G| = 12(p-1)$ and there is a bordered smooth epimorphism $\theta : \Gamma^* \rightarrow G$

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