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ON SOME EXPONENTIAL SUMS WITH EXPONENTIAL AND RATIONAL FUNCTIONS

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ABSTRACT. We study exponential sums of the form

$$\sum_{x=1}^{t}{}^{*} \exp(2\pi i(a\vartheta^x/p + f(x)/t))$$

where ϑ is an integer of multiplicative order t modulo a prime p, $f(X)$ is rational function modulo t and Σ^* indicates that the poles of f are excluded. The case of $f(X) = bX$ is well studied and has been considered in a number of works. For $f(X) = b/X$ these sums have recently been estimated by Bourgain and the author. Here we consider the general case of an arbitrary rational function f.

1. Introduction. For a prime p we denote by $\mathbf{F}_p$ the finite field of p elements, which we assume to be represented by the set $\{0, 1, \dots, p-1\}$. For an integer t we denote by $\mathbf{Z}_t$ the residue ring modulo t and by $\mathbf{Z}_t^*$ the group of units of $\mathbf{Z}_t$.

Let $\vartheta \in \mathbf{F}_p^*$ be of multiplicative order $t \geq 1$. Furthermore, for an integer $m > 0$, we put

$$\mathbf{e}_m(z) = \exp(2\pi i z/m),$$

and define the exponential sums

$$S_p(a; f) = \sum_{x \in \mathcal{X}_f} \mathbf{e}_p(a\vartheta^x)\mathbf{e}_t(f(x))$$

where $f(X)$ is rational function over $\mathbf{Z}_t$ and $\mathcal{X}_f$ is the set of $x \in \mathbf{Z}_t$ for which the denominator of $f(X)$ is a unit of $\mathbf{Z}_t$.

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