ON LIMITS OF SEQUENCES
OF HOLOMORPHIC FUNCTIONS

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ABSTRACT. We study functions which are the pointwise limit of a sequence of holomorphic functions. In one complex variable this is a classical topic, though we offer some new points of view and new results. Some novel results for solutions of elliptic equations will be treated. In several complex variables the question seems to be new, and we explore some new avenues.

1. Introduction. It is a standard and well-known fact from complex function theory (which appears to be due to Stieltjes [**23**], although see also Vitali's theorem in [**24**] and Weierstrass's complete works [**25**]) that, if $\{f_j\}$ is a sequence of holomorphic functions on a planar domain Ω and if the sequence converges *uniformly on compact subsets of* Ω, then the limit function is holomorphic on Ω. Certainly this result is one of several justifications for equipping the space of holomorphic functions on Ω with the compact-open topology (see also [**15**], where this point of view is developed in detail from the perspective of functional analysis).

Considerably less well known is the following result of Osgood [**19**]:

Theorem 1. *Let $\{f_j\}$ be a sequence of holomorphic functions on a planar domain Ω. Assume that the f_j converge pointwise to a limit function f on Ω. Then f is holomorphic on a dense, open subset V of Ω. The convergence is uniform on compact subsets of the dense, open set.*