THE CORONA THEOREM FOR MULTIPLIER ALGEBRAS ON WEIGHTED DIRICHLET SPACES

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ABSTRACT. We prove a corona theorem for infinitely many functions from the multiplier algebras on weighted Dirichlet spaces on the unit disk. In addition, explicit estimates on solutions are given.

In this paper we wish to extend the corona theorem on the multiplier algebra on a weighted Dirichlet space, $M(\mathcal{D}_\alpha)$, to infinitely many functions and obtain estimates on *corona problem* solutions. For a finite number of functions, the corresponding theorem is due to Tolokonnikov [8]. For infinitely many functions in $H^\infty(\mathbf{D})$, the corona theorem is due to Rosenblum [7] and to Tolokonnikov [8]. Our methods are in principle close to those of Rosenblum [7]. All of these efforts were made possible by Wolff's beautiful proof of Carleson's original corona theorem. (See [1]).

This paper uses the basic ideas first used by Trent [10] to solve the corresponding problem in Dirichlet space. Later these ideas were used in Costea, Sawyer and Wick [4] in their generalization to certain Besov spaces on the unit ball in $\mathbf{C}^n$. Their paper is much more general than the case considered here. However, their formidable estimates on integral operators do not seem to give explicit estimates on corona solutions, which only depend upon ε. Moreover, we can explicitly give a fixed space to which all our multipliers on weighted Dirichlet space extend, namely, the *harmonic weighted Dirichlet space*.

We will establish our notation. $\mathcal{D}_\alpha$ will denote the weighted Dirichlet space on the unit disk, $\mathbf{D}$. That is, for $\alpha \in \mathbf{R}$,