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SURVEY ARTICLE–GRAPHICAL REPRESENTATIONS OF FACTORIZATIONS IN COMMUTATIVE RINGS

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ABSTRACT. This article surveys the recent and active area of irreducible divisor graphs of commutative rings. Notable algebraic and graphical results are given, and alternate constructions for irreducible divisor graphs and higher dimensional analogs are explored.

1. Introduction and motivation. One of the main themes in the study of abstract algebra is the study of how elements of a commutative ring factor. From the fundamental theorem of arithmetic to Galois theory, the study of how elements can be written as a product of irreducible elements has been a fruitful one. In more recent years, this area of study has taken an interesting turn by looking to graph theory in search of a better understanding of factorization theory. This paper will survey recent results along these lines.

The idea of transforming a ring-theoretic question into a graph-theoretic one is certainly not a novel concept. For example, the past decade or so has seen a great deal of research being done in the area of zero-divisor graphs with the goal of trying to better understand the role of zero-divisors in a commutative ring. Let R be a commutative ring with nonzero identity, and let $Z(R)$ be its set of zero-divisors. The *zero-divisor graph* of R, denoted by $\Gamma(R)$, is the (undirected) graph with vertices $Z(R)^* = Z(R) \setminus \{0\}$, the nonzero zero-divisors of R, and for distinct $x, y \in Z(R)^*$, the vertices x and y are adjacent if and only if $xy = 0$ (cf. [**3**]). The interested reader can find an overview of zero divisor graphs in [**2**]. With the success of zero-divisor graphs in mind, we turn our attention to factorizations, for the most part, in integral domains.

Throughout, D will denote an integral domain, D^* will denote the nonzero elements of D, and $U(D)$ will denote the units of D. Recall

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