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COTORSION PAIRS IN C(R-Mod)

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ABSTRACT. In [**8**] Salce introduced the notion of a cotorsion pair $(\mathcal{A}, \mathcal{B})$ in the category of abelian groups. But his definitions and basic results carry over to more general abelian categories and have proved useful in a variety of settings. In this article we will consider complete cotorsion pairs $(\mathcal{C}, \mathcal{D})$ in the category $\mathbf{C}(R\text{-Mod})$ of complexes of left R-modules over some ring R. If $(\mathcal{C}, \mathcal{D})$ is such a pair, and if $\mathcal{C}$ is closed under taking suspensions, we will show when we regard $\mathbf{K}(\mathcal{C})$ and $\mathbf{K}(\mathcal{D})$ as subcategories of the homotopy category $\mathbf{K}(R\text{-Mod})$, then the embedding functors $\mathbf{K}(\mathcal{C}) \to \mathbf{K}(R\text{-Mod})$ and $\mathbf{K}(\mathcal{D}) \to \mathbf{K}(R\text{-Mod})$ have left and right adjoints, respectively. In finding examples of such pairs, we will describe a procedure for using Hoveys results in [**5**] to find a new model structure on $\mathbf{C}(R\text{-Mod})$.

1. Introduction. Let R be a ring, and let $\mathbf{C}(R\text{-Mod})$ denote the category of complexes of left R-modules. This category has enough injectives and projectives so we can compute derived functors. We let Ext^n denote the nth derived functor of Hom in the category of these complexes. We identify the elements of $\mathrm{Ext}^1(C, D)$ with the equivalence classes of short exact sequences

$$0 \longrightarrow D \longrightarrow U \longrightarrow C \longrightarrow 0$$

in $\mathbf{C}(R\text{-Mod})$.

If $C \in \mathbf{C}(R\text{-Mod})$, let $S(C)$ denote the suspension of the complex C. So $S(C)_n = C_{n+1}$ for all n, and the differential of $S(X)$ is d where d is the differential of C (with an appropriate change in subscripts). We then can define $S^k(C)$ for any $k \in \mathbf{Z}$. A class $\mathcal{C}$ of objects of $\mathbf{C}(R\text{-Mod})$ will be said to be closed under suspensions if $S^k(C) \in \mathcal{C}$ whenever $C \in \mathcal{C}$ and $k \in \mathbf{Z}$.

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