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SUBDIRECT PRODUCTS OF M^*-GROUPS

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ABSTRACT. A compact bordered Klein surface X of genus $g \geq 2$ has at most $12(g-1)$ automorphisms. A bordered surface for which the bound is attained is said to have maximal symmetry, and its full automorphism group is called an M^*-group. For M^*-groups G and H, we construct a subdirect product L of G and H that is an M^*-group. We show that there is a normal subgroup of G whose index is the same as the index of L in the direct product $G \times H$. This general result is specialized to give results about the index of the subdirect product L in the direct product $G \times H$ for M^*-groups G and H. Then we give a number of sufficient conditions for L to equal $G \times H$ and to conclude that the direct product is an M^*-group. For example, let G be an M^*-group that acts on a bordered Klein surface X. The elements of G that fix a boundary component of X form a dihedral subgroup of order $2q$. The number q is called an action index of G. If G and H have relatively prime action indices and one of them is perfect, then the direct product of G and H is an M^*-group.

1. Introduction.

A compact bordered Klein surface X of genus $g \geq 2$ has at most $12(g-1)$ automorphisms [**10**]. A bordered surface for which the bound is attained is said to have *maximal symmetry* [**8**]. The full automorphism group of a surface with maximal symmetry is called an M^*-group [**11**].

There are infinitely many M^*-groups, and some important groups are known to be M^*-groups. For example, all large alternating groups and all large symmetric groups are M^*-groups [**3**], as well as most of the groups $\mathrm{PSL}\,(2,q)$ [**19**]. In addition, there are constructions that give extensions of abelian groups by a particular M^*-group G; here see [**8**, Section 4]. These constructions do not produce a presentation of the extension, however. On the other hand, there is a construction that forms an M^*-group, with complete presentation, from a 2-generator group that admits an action of D_6, the smallest M^*-group [**14**].

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