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ON MINIMAL FINITE QUOTIENTS OF MAPPING CLASS GROUPS

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ABSTRACT. We prove that the minimal nontrivial finite quotient group of the mapping class group $\mathcal{M}_g$ of a closed orientable surface of genus g is the symplectic group $\mathrm{PSp}_{2g}(\mathbf{Z}_2)$, for $g = 3$ and 4 (this might remain true, however, for arbitrary genus $g > 2$). We also discuss some results for arbitrary genus g.

1. Introduction. It is an interesting, but in general difficult, problem to classify the finite quotients (factor groups) of certain geometrically interesting infinite groups. This becomes particularly significant if the group in question is perfect (has trivial abelianization) since in this case each finite quotient projects onto a minimal quotient which is a nonabelian finite simple group, and there is the well-known list of finite simple groups (always understood to be nonabelian in the sequel).

As an example, the finite quotients of the smallest volume Fuchsian triangle group of type (2,3,7) (two generators of orders two and three whose product has order seven) are the so-called Hurwitz groups, the groups of orientation-preserving diffeomorphisms of maximal possible order $84(g-1)$ of closed orientable surfaces of genus g. There is a rich literature on the classification of Hurwitz groups, and in particular of simple Hurwitz groups; the smallest Hurwitz group is the projective linear or linear fractional group $\mathrm{PSL}_2(7)$ of order 168, acting on Klein's quartic of genus three.

One of the most interesting groups in topology is the mapping class group $\mathcal{M}_g$ of a closed orientable surface $\mathcal{F}_g$ of genus g which is the group of orientation-preserving homeomorphisms of $\mathcal{F}_g$ modulo the subgroup of homeomorphisms isotopic to the identity; alternatively, it is the orientation-preserving subgroup of index two of the outer automorphism group $\mathrm{Out}\,(\pi_1(\mathcal{F}_g))$ of the fundamental group. It is well known that $\mathcal{M}_g$ is a perfect group, for $g \geq 3$ [**13**]. By abelianizing the

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