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BIFURCATION AND CHAOS IN A PULSED PLANKTON MODEL WITH INSTANTANEOUS NUTRIENT RECYCLING

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ABSTRACT. This paper deals with a pulsed plankton-nutrient interaction model consisting of phytoplankton, herbivorous zooplankton and dissolved limiting nutrient with general nutrient uptake functions and instantaneous nutrient recycling. We investigate the subsystem with nutrient and phytoplankton and study the stability of the periodic solutions, which are the boundary periodic solutions of the system. Stability analysis of the boundary periodic solution yields the invasion threshold of zooplankton. By use of standard techniques of bifurcation theory, we prove that, above this threshold, there are periodic oscillations in substrate: phytoplankton and zooplankton. Simple cycles may give way to chaos in a cascade of period-doubling bifurcations. Furthermore, by comparing bifurcation diagrams with different bifurcation parameters, we can see that the impulsive system shows two kinds of bifurcations, which are period-doubling and period-halving.

1. Introduction. Plankton blooms are a widespread phenomenon present in rivers, lakes and oceans all over the world. When large numbers of plankton are concentrated in one area, the color of the water surface changes. For example, in May 2007, a blue-green algae bloomed in Taihu Lake. Analysis of tremendous variations in an abundance of many planktonic communities is very important in aquatic ecosystems. Numerous plankton models have been constructed and studied by researchers in this area [**9, 11, 15**].

The chemostat is an interesting model frequently used for simulating a simple lake, with the input of limiting nutrients such as silica and phosphate from streams draining the surrounding watershed. See [**3,**

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