ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 42, Number 4, 2012

GLOBAL STABILITY OF TWO TUBERCULOSIS MODELS WITH TREATMENT AND SELF-CURE

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ABSTRACT. Two tuberculosis (TB) models with treatment and self-cure are investigated. It is assumed that the treated individuals may reenter either the latent compartment due to the remainder of Mycobacterium tuberculosis or the infectious compartment due to the treatment failure. On the other hand, infectious individuals with mild symptoms may reenter the latent compartment due to self-cure. After investigating a simple model with one latent compartment, a TB model with two latent compartments is studied to describe the slow and fast progression of the exposed individuals. The basic reproduction numbers of those two models are defined, and their epidemiological interpretation is given. Similar threshold dynamics is obtained for those two models: the disease dies out ultimately when the basic reproduction number is less than or equal to one; the unique endemic equilibrium is globally stable when the basic reproduction number is greater than one. The influence of treatment failure and self-cure on the basic reproduction number is also discussed.

1. Introduction. Tuberculosis (TB) is a bacterial disease caused by *Mycobacterium tuberculosis*, which is usually acquired through airborne infection from active TB cases. It is one of the most common infectious diseases and has been a leading cause of death in the world for centuries. About one-third of the world's population is infected with the TB bacillus [**1, 14**]. In comparison with other infective diseases, following primary infection, most of the individuals infected by TB (approximately 90 percent) stay in a latent stage, and only a small proportion of individuals develop the active TB. Therefore, introducing the latent compartment is necessary for modeling the transmission of TB.

Keywords and phrases. Tuberculosis, treatment, self-cure, equilibrium, global stability.

This work was partially supported by the Nature Science Foundation of Shaanxi Province of China (No. 2012JQ1019) and the Nature Science Foundation of China (No. 11071256).

Received by the editors on August 4, 2009, and in revised form on January 10, 2010.

DOI:10.1216/RMJ-2012-42-4-1367