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A CORRESPONDENCE BETWEEN THE ISOBARIC RING AND MULTIPLICATIVE ARITHMETIC FUNCTIONS

TRUEMAN MACHENRY AND KIEH WONG

ABSTRACT. We give a representation of the classical theory of multiplicative arithmetic functions (MF) in the ring of symmetric polynomials written on the isobaric basis. The representing elements are recursive sequences of Schur-hook polynomials evaluated on subrings of the complex numbers. Multiplicative arithmetic functions are units in the Dirichlet ring of arithmetic functions, and their properties can be described locally, that is, at each prime number p. Our representation is, hence, a local representation. This representation enables us to clarify and generalize classical results, e.g., the Busche-Ramanujan identity, as well as to give a richer structural description of the convolution group of multiplicative functions. It is a consequence of the representation that the MFs can be defined in a natural way on the negative powers of prime p which, in turn, leads to a natural extension of Schur-hook polynomials to negatively indexed Schur-hook polynomials.

0. Introduction. In this paper we give a representation of the classical theory of multiplicative arithmetic functions (MF) in the ring of symmetric polynomials. The Dirichlet ring of arithmetic functions $\mathcal{A}^*$ is well known to be a unique factorization domain (see Cashwell and Everett [**4**]). Its ring theoretic properties have been investigated in, e.g., Rearick [**29, 30**], Shapiro [**33**], Carroll and Gioia [**3**], MacHenry [**22**], MacHenry and Tudose [**25**]. The multiplicative arithmetic functions are units in this ring, and their properties can be described locally, that is, at each prime number p, (see, e.g., McCarthy [**27**], Sivaramakrishnan [**35**] and Vaidyanathswamy, [**36**]). It is this local behavior which we take advantage of to construct a representation in terms of a certain class of symmetric polynomials called *weighted isobaric polynomials* [**25**]. It is advantageous to use the *isobaric* basis as a basis for the

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