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PARABOLIC SUBGROUPS OF COXETER GROUPS ACTING BY REFLECTIONS ON CAT(0) SPACES

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ABSTRACT. We consider a cocompact discrete reflection group W of a CAT(0) space X. Then W becomes a Coxeter group. In this paper, we study an analogy between the Davis-Moussong complex $\Sigma(W, S)$ and the CAT(0) space X and show several analogous results about the limit set of a parabolic subgroup of the Coxeter group W.

1. Introduction and preliminaries. The purpose of this paper is to study the limit set of a parabolic subgroup of a reflection group of a CAT(0) space. A metric space (X, d) is called a *geodesic space* if for each $x, y \in X$, there exists an isometric embedding $\xi : [0, d(x, y)] \to X$ such that $\xi(0) = x$ and $\xi(d(x, y)) = y$ (such a ξ is called a *geodesic*). We say that an isometry r of a geodesic space X is a *reflection* of X, if

(1) r^2 is the identity of X,

(2) $\operatorname{Int} F_r = \varnothing$ for the fixed-point set F_r of r,

(3) $X \setminus F_r$ has exactly two convex components X_r^+ and X_r^-, and

(4) $rX_r^+ = X_r^-$ and $rX_r^- = X_r^+$,

where the fixed-point set F_r of r is called the *wall* of r. Let X_r^+ and X_r^- be the two convex connected components of $X \setminus F_r$, where X_r^+ contains a basepoint of X. An isometry group Γ of a geodesic space X is called a *reflection group*, if some set of reflections of X generates Γ.

Let Γ be a reflection group of a geodesic space X, and let R be the set of all reflections of X in Γ. Now we suppose that the action of Γ on X is proper, that is, $\{\gamma \in \Gamma \mid \gamma x \in B(x, N)\}$ is finite for any $x \in X$ and $N > 0$ (cf. [**2**, page131]). Then the set $\{F_r \mid r \in R\}$ is locally finite. Let C be a component of $X \setminus \bigcup_{r \in R} F_r$, which is called a *chamber*. Then $\Gamma C = X \setminus \bigcup_{r \in R} F_r$, $\Gamma \overline{C} = X$ and for each $\gamma \in \Gamma$, either $C \cap \gamma C = \varnothing$ or

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