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A DENSITY CONDITION FOR INTERPOLATION ON THE HEISENBERG GROUP

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ABSTRACT. We consider left invariant multiplicity free subspaces of $L^2(N)$ where N is the Heisenberg group. We prove a necessary and sufficient density condition in order that such subspaces possess the interpolation property with respect to a class of discrete subsets of N that includes the integer lattice. We exhibit a concrete example of a subspace that has interpolation for the integer lattice, and we also prove a necessary and sufficient condition for shift invariant subspaces to possess a singly-generated orthonormal basis of translates.

1. Introduction. Let $\mathcal{H}$ be a Hilbert space of continuous functions on a topological space X for which point evaluation $f \mapsto f(x)$ is continuous, let Γ be a countable discrete subset of X, and let p be the restriction mapping $f \mapsto f|_\Gamma$ on $\mathcal{H}$. We say that $(\mathcal{H}, \Gamma)$ is a (Parseval) sampling pair if p is a constant multiple of an isometry of $\mathcal{H}$ into $\ell^2(\Gamma)$, and if p is surjective then we say that $(\mathcal{H}, \Gamma)$ has the interpolation property. In the present work we are interested in sufficient conditions in order that a pair $(\mathcal{H}, \Gamma)$ has the interpolation property, where $\mathcal{H}$ is a Hilbert space of functions on the Heisenberg group. Sampling has been studied in related settings in [**2, 3, 5**]. In [**5**] the author obtains sampling sets for Paley-Wiener functions on stratified Lie groups, while some of the results in [**2**] provide a characterization of Parseval sampling pairs for left invariant subspaces of $L^2(G)$ where G is any locally compact unimodular Type I group, in terms of the notion of admissibility. The work of [**3**] also provides quite general sufficient conditions for sampling and includes ideas from both of the preceding articles. The statement made in [**2**, page 47, also Theorem 6.4 (ii)] that, on the Heisenberg group, left-invariant sampling spaces associated to lattices never have the interpolation property, is particularly pertinent to the present work.

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