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SURVEY ARTICLE: CONSEQUENCES OF SOME OUTERPLANARITY EXTENSIONS

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ABSTRACT. In this expository paper we revise some extensions of Kuratowski planarity criterion, providing a link between the embeddings of infinite graphs without accumulation points and the embeddings of finite graphs with some distinguished vertices in only one face. This link is valid for any surface and for some pseudosurfaces.

On the one hand, we present some key ideas that are not easily accessible. On the other hand, we state the relevance of infinite, locally finite graphs in practice and suggest some ideas for future research.

1. Introduction. The problem of extending Kuratowski's planarity criterion [**27**] to other surfaces different from the sphere, S^2, looks very difficult. This statement is shown by the fact that there has been little progress on this research from 1930 until 1979, when Archdeacon [**1**] and Glover, Huneke and Wang [**23**] determined the class of finite graphs which cannot be drawn in the projective plane, P_2, and which are minimal with this property under topological containment. These graphs were denoted by $T(P_2)$, and they found that $T(P_2)$ had 103 elements, in front of the only two in $T(S^2)$. However, the problem still remains open when dealing with any other compact surface different from S^2 and P_2.

More or less at the same time, infinite graphs constituted a relevant generalization of classic, finite graphs. The handling of these graphs presents substantial differences, but it is also possible to find common aspects. In fact, in this paper we establish a link between infinite graphs and finite graphs which verify some properties. Besides, we revise some characterizations of embeddings of infinite graphs, paying

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