ESSENTIAL NORM ESTIMATE OF
A COMPOSITION OPERATOR BETWEEN
BLOCH-TYPE SPACES IN THE UNIT BALL

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ABSTRACT. Let B_n be the unit ball of $\mathbf{C}^n$ and $\phi = (\phi_1, \ldots, \phi_n)$ a holomorphic self-map of B_n. Let $p,\ q > 0$, be an estimate of the essential norm of a bounded composition operator C_ϕ induced by ϕ between the p-Bloch space $\beta^p(B_n)$ and q-Bloch space $\beta^q(B_n)$ given in this paper, as well as the corresponding results between the little p-Bloch space $\beta_0^p(B_n)$. As a consequence, a necessary and sufficient condition for the composition operator C_ϕ to be compact from $\beta^p(B_n)$ (or $\beta_0^p(B_n)$) into $\beta^q(B_n)$ (or $\beta_0^q(B_n)$) is obtained.

1. Introduction. The class of all holomorphic functions with domain Ω will be denoted by $H(\Omega)$, where Ω is a bounded homogeneous domain in $\mathbf{C}^n$. Let ϕ be a holomorphic self-map of Ω, the composition operator C_ϕ induced by ϕ is defined by

$$(C_\phi f)(z) = f(\phi(z)),$$

for z in Ω and $f \in H(\Omega)$.

For $\Omega = B_n$ the unit ball of $\mathbf{C}^n$, Timoney [10] shows that $f \in H(B_n)$ is in the Bloch space $\beta(B_n)$ if and only if

$$\sup_{z \in B_n} (1 - |z|^2)|\nabla f(z)| < \infty.$$

This definition was the starting point for introducing the p-Bloch spaces [21].