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ONE HALF OF A MAXIMAL EMBEDDING DIMENSION NUMERICAL SEMIGROUP

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ABSTRACT. Let S be a numerical semigroup, and let p be a positive integer. Then $S/p = \{x \in \mathbf{N} \mid px \in S\}$ is also a numerical semigroup and, when $p = 2$, we say that $S/2$ is the one half of the numerical semigroup S. We characterize the numerical semigroups that are one half of a numerical semigroup with maximal embedding dimension. This characterization allows us to algorithmically determine, whether or not a given numerical semigroup is one half of a numerical semigroup with maximal embedding dimension.

1. Introduction. A numerical semigroup is a subset of $\mathbf{N}$ (here $\mathbf{N}$ denotes the set of nonnegative integers) that is closed under addition, contains the zero element and has finite complement in $\mathbf{N}$. Given $A \subseteq \mathbf{N}$, we will denote by $\langle A \rangle$ the submonoid of $(\mathbf{N}, +)$ generated by A, that is,

$$\langle A \rangle = \{\lambda_1 a_1 + \cdots + \lambda_n a_n \mid n \in \mathbf{N} \setminus \{0\},\ \lambda_1, \ldots, \lambda_n \in \mathbf{N}, a_1, \ldots, a_n \in A\}.$$

If $S = \langle A \rangle$, then we say that A is a system of generators of S. We say that A is a minimal system of generators of S if no proper subset of A generates S. It is well known (see for instance [**10**]) that every numerical semigroup admits a unique minimal system of generators, which has finitely many elements.

If S is a numerical semigroup and $\{n_1 < n_2 < \cdots < n_p\}$ is its minimal system of generators, then n_1 is called the multiplicity of S and we denote it by $\mathrm{m}(S)$. The positive integer p is the embedding dimension of S, and we denote it by $\mathrm{e}(S)$ (see [**3**]). It is easy to prove

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