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AN INVERSION FORMULA OF RADON TRANSFORM ON THE PRODUCT HEISENBERG GROUP

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ABSTRACT. Let $\mathbf{H}_1^n$ be the n-direct product of the Heisenberg groups $\mathbf{H}_1$, and let $\mathcal{P}$ be the affine group of $\mathbf{H}_1^n$. Then $\mathcal{P}$ has a natural unitary representation on $L^2(\mathbf{H}_1^n)$. In this article, we present the inversion of the Radon transform on the product Heisenberg group by using inverse wavelet transform. In addition, we characterize a subspace of $L^2(\mathbf{H}_1^n)$ such that the inversion formula of the Radon transform holds in the weak sense.

1. Introduction. Wavelet analysis has many applications in pure and applied mathematics. The concept of continuous wavelet transform is deeply related to the theory of square integrable group representations, see [**1**]. In this viewpoint the theory of continuous wavelet analysis on the Heisenberg group has been established, see [**4, 8**]. It is known that the Radon transform on $\mathbf{R}^n$ is a very useful analysis tool. Recently, we find that a lot of authors deal with the inversion formula of the Radon transform by using inverse wavelet transforms. The first result in this area is due to Holschneider who considered the classical Radon transform on the two-dimensional plane, see [**7**]. Rubin [**11–13**] extended the result to k-dimensional Radon transforms on $\mathbf{R}^n$ and totally geodesic Radon transforms on the sphere and hyperbolic space. Later, He, Liu, Nessibi and Trimèche studied analogous problems on the Heisenberg group, see [**3, 9**], and the other cases, see [**5, 6**]. At the same time, we also find that Radon transforms can be applied to estimate the regularity for solutions of nonlinear Schrödinger equations, see [**10**]. So, in this paper, we introduce partial Radon transforms on the product Heisenberg group and discuss the

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