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EQUIVALENCE OF REAL MILNOR FIBRATIONS FOR QUASI-HOMOGENEOUS SINGULARITIES

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ABSTRACT. We show that for real quasi-homogeneous singularities $f:(\mathbf{R}^m,0)\to(\mathbf{R}^2,0)$ with isolated singular point at the origin, the projection map of the Milnor fibration $S_\varepsilon^{m-1}\setminus K_\varepsilon\to S^1$ is given by $f/\|f\|$. Moreover, for these singularities the two versions of the Milnor fibration, on the sphere and on a Milnor tube, are equivalent. In order to prove this, we show that the flow of the Euler vector field plays and important role. In addition, we present, in an easy way, a characterization of the critical points of the projection $(f/\|f\|):S_\varepsilon^{m-1}\setminus K_\varepsilon\to S^1$.

1. Introduction. It is well known that in [**8**] Milnor studies certain fibrations associated to analytic functions in a neighborhood of an isolated critical point.

For real analytic mappings, he showed that given $f:(\mathbf{R}^m,0)\to(\mathbf{R}^p,0)$, $m\geq p\geq 2$, a real polynomial map germ whose derivative Df has rank p on a punctured neighborhood U of $0\in\mathbf{R}^m$, there exist $\varepsilon>0$ small and $\eta>0$, $0<\eta\ll\varepsilon\ll 1$, such that if we set $E:=B_\varepsilon^m(0)\cap f^{-1}(S_\eta^{p-1})$ (called a Milnor tube), then

$$f_{|_E}:E\longrightarrow S_\eta^{p-1}$$

is a smooth (locally trivial) fibre bundle, where $B_\varepsilon^m(0)$ is the closed ball of radius $\varepsilon>0$ and center 0 in $\mathbf{R}^m$, and S_η^{p-1} is the boundary of the closed ball $B_\eta^p(0)$ in $\mathbf{R}^p$.

This fibration induces a fibration given by

$$f_{|_{E^\circ}}:E^\circ\longrightarrow S_\eta^{p-1}, \tag{1}$$

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