

## THE TRACIAL RANK FOR CROSSED PRODUCTS BY FINITE GROUP ACTIONS

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**ABSTRACT.** We define the second tracial Rokhlin property for finite group actions. Let  $A$  be an infinite dimensional finite separable unital  $C^*$ -algebra and let  $\alpha : G \rightarrow \text{Aut}(A)$  be an action of a finite group  $G$  on  $A$  which has the second tracial Rokhlin property. Suppose that  $A$  is  $\alpha$ -simple. If  $A$  is an AF-algebra, we prove that the tracial rank of the crossed product  $A \rtimes_{\alpha} G$  is zero. If  $A$  is an AT-algebra with the (SP)-property, we prove that the tracial rank of the crossed product  $A \rtimes_{\alpha} G$  is no more than one.

**1. Introduction.** The concept of the tracial rank of  $C^*$ -algebras was introduced by Lin [11]. The purpose to introduce the rank was motivated by the Elliott program of classification of nuclear  $C^*$ -algebras.  $C^*$ -algebras with tracial rank no more than  $k$  for some  $k \in \mathbb{N}$  are  $C^*$ -algebras that can be approximated by  $C^*$ -subalgebras in  $\mathcal{I}^{(k)}$  in trace (or in “measure”). The  $C^*$ -algebras of real rank zero can be determined by K-theory and hence can be classified. For example, Lin proved that if a simple separable amenable unital  $C^*$ -algebra  $A$  has tracial rank zero and satisfies the Universal Coefficient theorem, then  $A$  is a simple AH-algebra with slow dimension growth and with real rank zero [12, 14].

The concept of the Rokhlin property in ergodic theory was adopted to the context of von Neumann algebras by Connes [2]. Then the Rokhlin property was adopted to the context of UHF-algebras by Herman and Ocneanu [7]. Rørdam [19] and Kishimoto [9] introduced the Rokhlin property to a much more general context of  $C^*$ -algebras. More recently, Phillips, Osaka and Lin et al. studied integer group or finite group actions which satisfy certain types of Rokhlin property on some  $C^*$ -

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