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BEST APPROXIMATION FORMULAS FOR THE DUNKL L^2-MULTIPLIER OPERATORS ON $\mathbf{R}^d$

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ABSTRACT. We study the Dunkl L^2-multiplier operators on $\mathbf{R}^d$, and we give for them Calderón's reproducing formulas and best approximation formulas using the theory of Dunkl transform and reproducing kernels.

1. Introduction. The Dunkl operators $\mathcal{D}_j$; $j = 1, \ldots, d$, on $\mathbf{R}^d$, are parameterized differential-difference operators [**2**], acting on some Euclidean space. These operators extend the usual partial derivatives by additional reflection terms and give rise to generalizations of many multi-variable analytic structures like the exponential function, the Fourier transform and the standard convolution [**3, 4, 7, 15**]. During the last decade, such operators have found considerable attention in various areas of mathematics and mathematical physics [**3, 4, 7, 9**]. They allow the development of Dunkl L^2-multiplier operators on $\mathbf{R}^d$ from classical theory of Fourier analysis (see [**6, 11, 12, 17**]).

The Dunkl analysis, with respect to the multiplicity function k, concerns the Dunkl operators $\mathcal{D}_j$, Dunkl transform $\mathcal{F}_k$ and Dunkl convolution $*_k$ on $\mathbf{R}^d$. In the limit case $k = 0$; $\mathcal{D}_j$, $\mathcal{F}_k$ and $*_k$ agree with the partial derivatives ∂_j, Fourier transform $\mathcal{F}$ and standard convolution $*$, respectively.

Let m be a function in the Lebesgue space $L^2(\mathbf{R}^d, w_k(x)dx)$, where w_k is a positive weight function on $\mathbf{R}^d$ which will be defined later in Section 2. We define the Dunkl L^2-multiplier operators on $\mathbf{R}^d$, for regular functions f, by

$$T_{k,m,a}f(x) := \mathcal{F}_k^{-1}[m_a\mathcal{F}_k(f)](x), \quad a > 0,$$

where m_a is the function given by

$$m_a(x) = m(ax).$$

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