A DIRICHLET ANALOGUE OF THE FREE MONOGENIC INVERSE SEMIGROUP VIA MÖBIUS INVERSION

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ABSTRACT. The M"obius inversion formula of the free monogenic inverse semigroup is represented by the M"obius function for Cauchy product. In this short note we describe a Dirichlet analogue of this inverse semigroup.

1. Introduction. The Cauchy product and the Dirichlet product are familiar convolutions of arithmetical functions. The corresponding M"obius functions $\mu_C$ and $\mu_D$ (as convolution inverses of the zeta function) are the following ones:

$$\mu_C(n) = \begin{cases} 
1 & \text{if } n = 0 \\
-1 & \text{if } n = 1 \\
0 & \text{if } n > 1
\end{cases}$$

$$\mu_D(n) = \begin{cases} 
1 & \text{if } n = 1 \\
(-1)^k & \text{if } n = p_1 \cdots p_k \text{ where } p_i \text{ are distinct primes} \\
0 & \text{if } p^2 \mid n \text{ for some prime } p
\end{cases}$$

The reduced standard division category $C_F(S)$ of an inverse monoid $S$ relative to an idempotent transversal $F$ of the $D$-classes of $S$ with $1 \in F$, $(\text{Ob } C_F(S) = F; \text{Hom}(e,f) = \{(s,e) \in S \times F | s^{-1}s \leq e, ss^{-1} = f\}$; $e \xrightarrow{(s,e)} f \xrightarrow{(t,f)} g = e \xrightarrow{(t,f)(s,e)^{-1}(t_s,e)} g$ is a M"obius category if and only if $S$ is combinatorial and $(E(S), \leq)$ is locally finite (see [10]). If an inverse semigroup $S$ is without identity it may be converted into an inverse monoid by adjoining an identity. We call the M"obius inversion formula of such M"obius category $C_F(S)$, the M"obius inversion formula of the inverse monoid (semigroup) $S$. In [11] there are given M"obius inversion formulas of some inverse semigroups $S$:

$$F, G : \mathbb{N} \to \mathbb{C}, \begin{array}{ll} 
F(n) = \sum_{i=0}^{n} G(i) & \iff G(n) = \sum_{i=0}^{n} \mu_C(n-i)F(i)
\end{array}$$