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A DIRICHLET ANALOGUE OF THE FREE MONOGENIC INVERSE SEMIGROUP VIA MÖBIUS INVERSION

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ABSTRACT. The Möbius inversion formula of the free monogenic inverse semigroup is represented by the Möbius function for Cauchy product. In this short note we describe a Dirichlet analogue of this inverse semigroup.

1. Introduction. The Cauchy product and the Dirichlet product are familiar convolutions of arithmetical functions. The corresponding Möbius functions μ_C and μ_D (as convolution inverses of the zeta function) are the following ones:

$$\mu_C(n) = \begin{cases} 1 & \text{if } n = 0 \\ -1 & \text{if } n = 1 \\ 0 & \text{if } n > 1 \end{cases} \tag{1.1}$$

$$\mu_D(n) = \begin{cases} 1 & \text{if } n = 1 \\ (-1)^k & \text{if } n = p_1 \cdots p_k \text{ where } p_i \text{ are distinct primes} \\ 0 & \text{if } p^2 \mid n \text{ for some prime } p \end{cases} \tag{1.2}$$

The reduced standard division category $C_F(S)$ of an inverse monoid S relative to an idempotent transversal F of the $D-$classes of S with $1 \in F$, $(\operatorname{Ob} C_F(S) = F$; $\operatorname{Hom}(e,f) = \{(s,e) \in S \times F | s^{-1}s \leq e, ss^{-1} = f\}$; $e \overset{(s,e)}{\longrightarrow} f \overset{(t,f)}{\longrightarrow} g = e \overset{(t,f)(s,e)=(ts,e)}{\longrightarrow} g)$ is a Möbius category if and only if S is combinatorial and $(E(S), \leq)$ is locally finite (see [**10**]). If an inverse semigroup S is without identity it may be converted into an inverse monoid by adjoining an identity. We call the Möbius inversion formula of such Möbius category $C_F(S)$, the Möbius inversion formula of the inverse monoid (semigroup) S. In [**11**] there are given Möbius inversion formulas of some inverse semigroups S:

$$F, G : \mathbf{N} \longrightarrow \mathbf{C},\ F(n) = \sum_{i=0}^{n} G(i) \Leftrightarrow G(n) = \sum_{i=0}^{n} \mu_C(n-i) F(i) \tag{1.3}$$

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