ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 41, Number 5, 2011

THE FIRST COHOMOLOGY GROUP OF MODULE EXTENSION BANACH ALGEBRAS

A.R. MEDGHALCHI AND H. POURMAHMOOD-AGHABABA

ABSTRACT. Let A be a Banach algebra and X a Banach A-bimodule. Then $\mathcal{S} = A \oplus X$, the l_1-direct sum of A and X becomes a module extension Banach algebra when equipped with the algebra product $(a,x).(a',x') = (aa', ax' + xa')$. In this paper we compute the first cohomology group $H^1(\mathcal{S},\mathcal{S})$ for module extension Banach algebras $\mathcal{S}$. Also we obtain results on n-weak amenability of commutative module extension Banach algebras. We have shown that there are many different examples of non-n-weakly amenable Banach algebras.

1. Introduction. Let A be a Banach algebra and X a Banach A-bimodule. A derivation from A into X is a bounded linear map satisfying

$$D(ab) = a.(Db) + (Da).b \quad (a, b \in A).$$

For each $x \in X$ we denote by ad_x the derivation $D(a) = a.x - x.a$, for all $a \in A$, called an inner derivation. We denote by $Z^1(A,X)$ the space of all derivations from A into X, and by $B^1(A,X)$ the space of all inner derivations from A into X. The first cohomology group of A with coefficients in X, denoted by $H^1(A,X)$, is the quotient space $Z^1(A,X)/B^1(A,X)$. This first cohomology group of a Banach algebra gives vast information about the structure of A. If X is a Banach A-bimodule, X^* (the dual space of X) is an A-bimodule as usual. Let $n \in \mathbf{N}$, the set of non-negative integers. A Banach algebra A is called amenable if $H^1(A,X^*) = 0$ for every A-bimodule X. A Banach algebra A is called n-weakly amenable (weakly amenable in case $n = 1$) if $H^1(A,A^{(n)}) = 0$, where $A^{(n)}$ is the n-th dual space of A and $A^{(0)} = A$ (cf. [**3**]). In [**5, 6**] the authors have calculated the first cohomology group of a class of Banach algebras which they called *triangular Banach algebras*.

2010 AMS *Mathematics subject classification.* Primary 16E40, 46H25.
Received by the editors on June 24, 2008, and in revised form on January 26, 2009.

DOI:10.1216/RMJ-2011-41-5-1639