ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 41, Number 4, 2011

DEGREE k LINEAR RECURSIONS mod (p) AND NUMBER FIELDS

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ABSTRACT. Linear recursions of degree k are determined by evaluating the sequence of generalized Fibonacci polynomials, $\{F_{k,n}(t_1, \dots, t_k)\}$ (isobaric reflects of the complete symmetric polynomials) at the integer vectors $(t_1, \dots, t_k)$. If $F_{k,n}(t_1, \dots, t_k) = f_n$, then

$$f_n - \sum_{j=1}^{k} t_j f_{n-j} = 0,$$

and $\{f_n\}$ is a linear recursion of degree k. On the one hand, the periodic properties of such sequences modulo a prime p are discussed and are shown to be related to the prime structure of certain algebraic number fields; for example, the arithmetic properties of the period are shown to characterize ramification of primes in an extension field. On the other hand, the structure of the semi-local rings associated with the number field is shown to be completely determined by Schur-hook polynomials.

1. Introduction. A sequence $\{f_n\}$ is a *linear recursion of degree* k, denoted by $[t_1, \dots, t_k]$, if, given a sequence of integers $t_1, \dots, t_k$, the following equation is satisfied for all $n \in \mathbf{Z}$:

$$f_n - \sum_{j=1}^{k} t_j f_{n-j} = 0. \tag{1.1}$$

In this paper we shall discuss the periodic nature of such sequences and the periodic nature of such sequences modulo primes. In particular, we characterize those k-linear sequences which are periodic, and those which are periodic modulo a prime. While we believe that these

Keywords and phrases. Symmetric polynomials, Schur polynomials, linear recursions, number fields.
Received by the editors on May 9, 2007, and in revised form on November 6, 2008.
DOI:10.1216/RMJ-2011-41-4-1303