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FILIFORM NILSOLITONS OF DIMENSION 8

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ABSTRACT. A Riemannian manifold (M, g) is said to be *Einstein* if its Ricci tensor satisfies $\operatorname{ric}(g) = cg$, for some $c \in \mathbf{R}$. In the homogeneous case, a problem that is still open is the so-called *Alekseevskii conjecture*. This conjecture says that any homogeneous Einstein space with negative scalar curvature (i.e., $c < 0$) is a *solvmanifold*: a simply connected solvable Lie group endowed with a left invariant Riemannian metric. The aim of this paper is to classify Einstein solvmanifolds of dimension 9 whose nilradicals are *filiform* (i.e., $(n-1)$-step nilpotent and n-dimensional).

1. Introduction. A Riemannian manifold (M, g) is said to be *Einstein* if its Ricci tensor satisfies $\operatorname{ric}(g) = cg$, for some $c \in \mathbf{R}$. Einstein metrics are often considered as the nicest, or most distinguished metrics on a given differentiable manifold (see for instance [**1**, Introduction]).

In the homogeneous case, a problem that is still open is the so-called *Alekseevskii conjecture* (see [**1**, 7.57]). This conjecture says that any homogeneous Einstein space with negative scalar curvature (i.e., $c < 0$) is a *solvmanifold*: a simply connected solvable Lie group endowed with a left invariant Riemannian metric. It is important to note that, nowadays, it is still unknown which solvable Lie groups admit a left invariant Einstein metric.

In [**6**], Lauret has proved that any Einstein solvmanifold S is *standard* (i.e., $[\mathfrak{a}, \mathfrak{a}] = 0$, where $\mathfrak{a} := [\mathfrak{s}, \mathfrak{s}]^{\perp}$, $\mathfrak{s}$ the Lie algebra of S). The study of standard Einstein solvmanifolds has been reduced to the rank-one case, that is, $\dim \mathfrak{a} = 1$, where strong structural and uniqueness results are well known (see [**3**]).

A nilpotent Lie algebra $\mathfrak{n}$ is called an *Einstein nilradical* if it is the nilradical (i.e., maximal nilpotent ideal) of the Lie algebra of an Einstein solvmanifold. It is proved in [**4**] that $\mathfrak{n}$ is an Einstein nilradical

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