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SELECTION THEOREMS AND MINIMAL MAPPINGS IN A CLUSTER SETTING

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ABSTRACT. The paper introduces a generalized concept of the cluster points of multi-function as a unified approach to the study of selection theorems, closed graph theorems and minimal multi-functions. The crucial notion is that of generalized lower and upper quasi continuity with respect to a given cluster system.

1. Introduction. The basic notion of this paper is a cluster process with respect to a given nonempty system $\mathcal{E}$ of the nonempty subsets of a topological space X, and the investigation is focused on the mutual connection between the original multifunction F and its resultant cluster multi-function $\mathcal{E}_F$. This concept enables us to describe the techniques for finding selections and to study the properties of multi-functions from many points of view. The main results of the paper concern the connection between a semi-$\mathcal{E}$-continuous multi-function and its $\mathcal{E}$-cluster multi-function. Further, the theorems concerning closed graph, minimality and the existence of quasi continuous selection for u-$\mathcal{E}$-continuous multi-functions are given.

2. Basic symbols and terminology. In the sequel X, Y are topological spaces, the symbols $\overline{A}$, A°, $\mathbf{N}$ and $\mathbf{R}$, respectively, denote the closure of a set A, the interior of A, the natural numbers $\{0, 1, 2, \dots\}$, and the reals with their usual topology. A compact/locally compact space is understood as a T_2 space, hence any locally compact space is regular (even completely regular).

A set S is quasi-open (terminology by [**20**] where a connection between quasi-continuity and the quasi-open sets (also called semi-open) was accented), if for any open set H intersecting S there is a

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