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AN EQUATION RELATED TO TWO-SIDED CENTRALIZERS IN PRIME RINGS

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ABSTRACT. The purpose of this paper is to prove the following result. Let m and n be positive integers, and let R be a prime ring with $\operatorname{char}(R) = 0$ or $m+n+1 \leq \operatorname{char}(R)$. Let $T : R \to R$ be an additive mapping satisfying the relation $T(x^{m+n+1}) = x^m T(x) x^n$ for all $x \in R$. In this case T is a two-sided centralizer.

This research is a continuation of our work [**6**]. Throughout, R will represent an associative ring with center $Z(R)$. Given an integer $n > 1$, a ring R is said to be n-torsion free, if for $x \in R$, $nx = 0$ implies $x = 0$. Recall that a ring R is prime if for $a, b \in R$, $aRb = (0)$ implies that either $a = 0$ or $b = 0$, and is semiprime in case $aRa = (0)$ implies $a = 0$. An additive mapping $T : R \to R$ is called a left centralizer in case $T(xy) = T(x)y$ holds for all pairs $x, y \in R$. For a semiprime ring R all left centralizers are of the form $T(x) = qx$ for all $x \in R$, where q is an element of Martindale right ring of quotients Q_r of R (see [**3**, Chapter 2]). In case R has the identity element $T : R \to R$ is a left centralizer if and only if T is of the form $T(x) = ax$ for all $x \in R$ and some fixed element $a \in R$. The definition of a right centralizer should be self-explanatory. An additive mapping T is called a two-sided centralizer in case T is a left and a right centralizer. In case $T : R \to R$ is a two-sided centralizer, where R is a semiprime ring with extended centroid C, then there exists an element $\lambda \in C$ such that $T(x) = \lambda x$ for all $x \in R$ (see [**3**, Theorem 2.3.2]). An additive mapping $T : R \to R$ is called a left (right) Jordan centralizer in case $T(x^2) = T(x)x$ ($T(x^2) = xT(x)$) holds for all $x \in R$. Zalar [**14**] has proved that any left (right) Jordan centralizer on a 2-torsion free semiprime ring is a left (right) centralizer. Molnár [**8**] has proved that in case we have an additive mapping $T : A \to A$, where A is a semisimple H^*-algebra, satisfying the relation $T(x^3) = T(x)x^2$ ($T(x^3) = x^2T(x)$) for all $x \in A$, then T is a left (right) centralizer. For

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