

## STOCHASTIC EXTINCTION AND RUNAWAY GROWTH IN DISCRETE BIOLOGICAL MODELS

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**ABSTRACT.** The consequences of global warming include predictions of 300,000 deaths per year, as well as the extinction of 100–500 species of birds per degree centigrade warming. Warming effects are also thought to play a role in runaway growth of other species, e.g., the quagga mussel invasion of Lake Mead. The mechanisms underlying runaway growth or extinction are poorly understood. We investigate these mechanisms in a discrete population equation which models the effects of environmental fluctuations of the population growth rate. The model predicts extinction when  $E(\ln(l))$ , the geometric mean of the population growth rate, decreases below zero, and runaway growth when  $E(\ln(l)) > 0$ . A major challenge is to estimate realizations at specific generation numbers,  $n$ , during runaway growth, or extinction, type events. Thus, we our main focus is to derive dynamic bounds which estimate realizations, at each  $n \geq 1$ , during the entire course of such events. These estimates are illustrated with examples. In particular, we give new insights into the dynamics of the present day ongoing Kenyan lion extinction.

**1. Background and goals.** At the close of the 2009 Copenhagen Climate Change conference it became increasingly clear that the nations of planet earth cannot agree on a carbon emission policy that is sufficient to stem the effects of global warming. As a result, there is a substantial probability of a 4–6 degree (Celsius) global rise in temperature by the year 2100. The consequence of such a temperature increase includes the threat of extinction of a multitude of biological species, including the human race. Already, the 2009 Global Humanitarian Forum report on the human impact of climate change and global warming estimates that “300,000 lives are lost each year due to climate change, and that nine out of ten are related to environmental degradation” [8]. A 2008 study [18] predicts that “Worldwide, every degree centigrade of warming projects a nonlinear increase of bird extinctions of 100–500 species.” An August, 2009 report by the Kenya Wildlife Service states

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