ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 41, Number 2, 2011

NAGUMO CONDITIONS AND SECOND-ORDER QUASILINEAR EQUATIONS WITH COMPATIBLE NONLINEAR FUNCTIONAL BOUNDARY CONDITIONS

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Dedicated to the memory of Lloyd Jackson

ABSTRACT. We establish existence results for solutions to nonlinear functional boundary value problems for nonlinear second-order ordinary differential equations assuming there are lower and upper solutions and the right side satisfies a Nagumo growth bound. Our results contain as special cases many results for the p- and ϕ-Laplacians as well as many results where the boundary conditions depend on n-points or even functionals.

1. Introduction.

$$-\frac{d}{dt}\varphi(t,x,x(t),x'(t)) = f(t,x,x(t),x'(t)), \quad \text{for a.e. } t\in[0,1], \tag{1}$$

subject to general functional boundary conditions of the form

$$G(x(0),x(1),x,x'(0),x'(1)) = (0,0), \tag{2}$$

where $\varphi \in C([0,1]\times C[0,1]\times \mathbf{R}^2)$, $f : [0,1]\times C[0,1]\times \mathbf{R}^2 \to \mathbf{R}$ satisfies the Carathédory conditions and $G\in C(\mathbf{R}^2\times C[0,1]\times\mathbf{R}^2;\mathbf{R}^2)$. Our assumptions on φ and f are due to Cabada and Pouso [**7**]. By a solution x we mean a function $x\in C^1[0,1]$ satisfying (2) such that $\varphi(t,x,x(t),x'(t))$ is absolutely continuous and satisfies (1) almost everywhere on $[0,1]$. We assume that there are ordered lower and upper solutions, α and β, respectively, for (1) and that the functional boundary conditions are compatible, in a sense defined below. The assumptions on φ are sufficiently general to apply to

$$(r(t)x'+q(t)x)' = f(t,x,x(t),x'(t))$$

2010 AMS *Mathematics subject classification.* Primary 34B10, 34B15.
Received by the editors on June 20, 2010.

DOI:10.1216/RMJ-2011-41-2-573