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BOUNDARY VALUE PROBLEMS FOR SINGULAR ELLIPTIC EQUATIONS

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To the memory of Lloyd K. Jackson

ABSTRACT. We study the existence of positive solutions to singular elliptic boundary value problems involving the p-Laplace operator. We establish a sub-supersolution theorem and use an eigenfunction of the p-Laplacian to construct sub- and super-solutions. Our assumptions on the singular term are more relaxed than in some previous papers, even for the case $p = 2$, as we allow for non-monotone singular terms with blowup controlled by a power. We also allow for a parameter dependent term and study how its growth affects our existence result.

1. Introduction. Let Ω be a smooth bounded domain in $\mathbf{R}^N$, $N \geq 1$, and $p > 1$. We are interested in the following singular elliptic problem

$$\begin{cases} -\Delta_p u = ag(u) + \lambda h(u) & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where

$$\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$$

is the p-Laplace operator; λ is a nonnegative parameter;

$$a : \Omega \longrightarrow [1, \infty)$$

is in $L^\infty(\Omega)$;

$$g : (0, \infty) \longrightarrow \mathbf{R}$$

is continuous and satisfies

$$\lim_{s \to 0^+} g(s) = \infty;$$

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