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OSCILLATION OF nTH ORDER SUPERLINEAR DYNAMIC EQUATIONS ON TIME SCALES

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We dedicate this paper to the memory of Lloyd K. Jackson

ABSTRACT. Consider the following nth order superlinear dynamic equation

$$x^{\Delta^n}(t) + p(t)x^\alpha(\sigma(t)) = 0, \quad \alpha > 1,$$

where $p \in C_{rd}(\mathbf{T}, \mathbf{R}^+)$, and $\mathbf{T}$ is an isolated time scale, α is a ratio of odd positive integers. We obtain an analog of the Kiguradze-Ličko-Švec-type oscillation theorem for this dynamic equation. As an application, we obtain

(i) when n is even, every solution $x(k)$ of the difference equation

$$\Delta^n x(k) + p(k)x^\alpha(k+1) = 0,$$

where $p(k) \geq 0$ and $\alpha > 1$ is oscillatory if and only if

$$\sum_{k=1}^{\infty} (k+1)^{n-1} p(k) = \infty.$$

(ii) when n is odd, every solution $x(k)$ of this difference equation is either oscillatory or $\lim_{k\to\infty} x(k) = 0$ if and only if the above sum is infinite.

1. Introduction. Consider the following nth order superlinear dynamic equation on a time scale

$$x^{\Delta^n}(t) + p(t)x^\alpha(\sigma(t)) = 0, \quad \alpha > 1, \tag{1.1}$$

where $p \in C_{rd}(\mathbf{T}, \mathbf{R}^+)$, $\mathbf{T}$ is a time scale, and α is a ratio of odd positive integers.

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