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GPU-BASED METHODS FOR EXPLORING PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS

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ABSTRACT. The mathematical investigation of solutions to a parabolic partial differential equation (PDE) can be complemented by numerically computing the solutions in order to obtain insight about their qualitative structure. The numerical methods are relatively slow on central processing units (CPUs), making it difficult to obtain rapid feedback about the solution over time. Graphics processing units (GPUs) are efficient hardware for massively parallel computations and provide rapid feedback. The numerical methods for parabolic PDEs map naturally onto the GPU but differ from the same methods implemented on a CPU. We describe the ideas for GPU-based solvers and illustrate them using parabolic PDEs that arise in combustion models.

1. Introduction. The concept of using numerical solutions to PDEs in order to gain insight about the qualitative structure of the theoretical solutions is not new, of course. In fact, several of the results mentioned in [**2**] were motivated by numerical experiments; in particular, they motivated the study of solution profiles and the relationship of their shape to bifurcation diagrams [**3**] and for developing generalized maximum principles [**1, 7**].

At that time (25 years ago), the experiments were performed on hardware that included an Intel 80486 CPU with an 80487 floating-point coprocessor and an Enhanced Graphics Adapter card. By today's standards, such hardware might as well be displayed in a museum about ancient computing devices. Back then it was enough to give some idea about solutions to the PDEs but was not suitable for visualization in interactive time let alone real-time.

Current generation hardware is much different from that of 25 years ago. CPUs now have multiple units of execution called cores. GPUs were invented to support the demands of consumers wanting more realistic graphics in computer video games. Although the evolution of CPUs and GPUs has been driven by consumer entertainment,

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