ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 41, Number 2, 2011

LINEAR SYSTEMS OF FRACTIONAL NABLA DIFFERENCE EQUATIONS

FERHAN M. ATICI AND PAUL W. ELOE

ABSTRACT. In this paper we shall consider a linear system of fractional nabla difference equations with constant coefficients. We shall construct the fundamental matrix for the homogeneous system and the causal Green's function for the nonhomogeneous system. We employ transform methods and series methods and we illustrate analogies with classical first order differential or difference equations. We shall close the paper with an asymptotic result that follows from the analysis of a half-order nabla difference equation.

1. Introduction. In this article we shall provide an introductory study to a system of fractional difference equations of the form

$$\nabla_0^\nu y(t) = Ay(t) + f(t), \quad t = 1, 2, \dots, \tag{1.1}$$

where A denotes an $n \times n$ matrix with constant entries, y and f denote $n-$vector valued functions and $0 < \nu < 1$. The operator ∇_a^ν, a Riemann-Liouville fractional difference, is defined as follows. If $\mu > 0$, define the μth fractional sum by

$$\nabla_a^{-\mu} y(t) = \sum_{s=a}^{t} \frac{(t-\rho(s))^{\overline{\mu-1}}}{\Gamma(\mu)} y(s)$$

where $\rho(s) = s - 1$ and the raising factorial power function is defined by $t^{\overline{\alpha}} = \Gamma(t+\alpha)/\Gamma(t)$. Then, if $0 \le n-1 < \nu \le n$, define the νth fractional difference (a Riemann-Liouville fractional difference) by $\nabla^\nu y(t) = \nabla^n \nabla^{\nu-n} y(t)$ where ∇^n denotes the standard nth order backward difference. So, in this article, with $0 < \nu < 1$, $\nabla_0^\nu y(t) = \nabla \nabla_0^{\nu-1} y(t)$. Anastassiou [**3**] has introduced the study of nabla fractional calculus in the case of the Caputo fractional difference.

2010 AMS *Mathematics subject classification.* Primary 39A12, 34A25, 26A33.
Keywords and phrases. Discrete fractional calculus, discrete Mittag-Leffler function.
Received by the editors on May 24, 2010.
DOI:10.1216/RMJ-2011-41-2-353