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ON ORTHOGONAL CHORDS IN NORMED PLANES

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ABSTRACT. It is known that a convex plate of diameter 1 in the Euclidean plane is of constant width 1 if and only if any two perpendicular intersecting chords have total length at least 1. We show that, in general, this result cannot be extended to normed (or Minkowski) planes when the type of orthogonality is defined in the sense of Birkhoff. Inspired by this, we present also further results on intersecting chords in normed planes that are orthogonal in the sense of Birkhoff and in the sense of James.

1. Introduction. A convex body in Euclidean space $\mathbf{R}^d$, $d \geq 2$, is called *of constant width* if the distance between any two parallel supporting hyperplanes is constant. There is a large variety of noncircular and, for $d \geq 3$, nonspherical convex bodies of constant width (see, e.g., the surveys [**6, 8**]). The most famous one is the *Reuleaux triangle* in the Euclidean plane. It is representable as the intersection of three circles of radius $r > 0$ which are centered at the vertices of an equilateral triangle of side-length r.

The notion of convex body of constant width is naturally extended to normed linear (or Minkowski) spaces, and so one can also define Minkowskian analogues of Reuleaux triangles (see [**6, 12, 13**] and [**14**, subsection 4.2]).

Makai and Martini [**11**] proved that in the Euclidean plane a convex body of diameter 1 is of constant width 1 if and only if any two perpendicular intersecting chords of it have total length greater than or equal to 1. Soltan has posed the question of characterizing the Minkowski geometries for which the analogue of this result holds (see [**11, 13**]). In this case, instead of perpendicularity, we consider orthogonality in the sense of Birkhoff.

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