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PROJECTIVE OPERATOR SPACES, ALMOST PERIODICITY AND COMPLETELY COMPLEMENTED IDEALS IN THE FOURIER ALGEBRA

BRIAN FORREST

ABSTRACT. We will show how projective operator spaces arise naturally as spaces of almost periodic functions. In particular, we will show that a locally compact group is compact if and only if its Fourier-Stieltjes algebra (or equivalently its Fourier algebra) is projective as an operator space. From this we see that if K is a compact subgroup of G, then the ideal $I(K)$ is completely complemented in $A(G)$.

1. Introduction. The notion of an operator space as a quantized analog of a Banach space has gained considerable attention of late. Four of the principal objects of abstract harmonic analysis; the group algebra $L^1(G)$, the measure algebra $M(G)$, the Fourier algebra $A(G)$ and Fourier-Stieltjes algebra $B(G)$ of a locally compact group, are operator spaces by virtue of being preduals of von Neumann algebras. In commutative harmonic analysis, the fact that these algebras are preduals of von Neumann algebras is seldom if ever considered. Generally speaking, the same is true of the study of the algebras $L^1(G)$ and $M(G)$ even for noncommutative groups. However, for the Fourier and Fourier-Stieltjes algebras of noncommutative groups one can not stray far from the world of operator algebras. For this reason, it seems very natural to include the operator space structure as part of our working environment. In this paper, we will show how for certain types of problems concerning the Fourier and Fourier-Stieltjes algebras this additional structure is essential. We will also illustrate how the operator space structure was actually playing a fundamental role even in the study of commutative groups or in the study of $L^1(G)$ and $M(G)$ without our noticing.

To begin with, we note that the group algebra and the measure algebra are in a strong sense dual objects of the Fourier and Fourier

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