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ON THE CONNECTIVITY OF ATTRACTORS OF ITERATED FUNCTION SYSTEMS

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ABSTRACT. The aim of the paper is to give sufficient and necessary conditions when the components of a fixed order of the attractor of an iterated function system are connected sets. By a component of order n of the attractor of an IFS we understand the image of the attractor through the composition of n functions from the IFS. This gives sufficient conditions when the attractor of an iterated function system is a finite union of connected sets. If these conditions are fulfilled the attractor of the iterated function system will be locally arcwise connected and every connected component of the attractor of the iterated function system will be arcwise connected.

1. Introduction. We start with a brief presentation of iterated function systems, IFSs for short. We will also fix the notations. Iterated function systems were conceived in the present form by Hutchinson [**5**] and popularized by Barnsley [**2**] and are one of the most common and most general ways to generate fractals. Many of the important examples of functions and sets with special and unusual properties turn out to be fractal sets, and a great portion of them are attractors of IFSs. There is a current effort to extend the classical Hutchinson's framework to more general spaces and infinite iterated function systems (IIFSs) or, more generally, to multifunction systems and to study them (see for example [**1, 7–10**]). A recent such example can be found in [**8**] where the Lipscomb's space, which was an important example in dimension theory, can be obtained as an attractor of an IIFS defined in a very general setting. In this setting the attractor can be a closed and bounded set in contrast with the classical theory where only compact sets are considered. Although fractal sets are defined with measure theory, being sets with noninteger Hausdorff dimension [**3, 4**], it turns out that they have interesting topological properties as we can see from

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