

HARMONICITY ON COSYMPLECTIC MANIFOLDS

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ABSTRACT. We prove that a (φ, J) -holomorphic map from a compact cosymplectic manifold to a Kähler manifold is not only a harmonic map but also an energy minimizer in its homotopy class. We also prove a converse result.

1. Introduction. Combining both global and local aspects and borrowing both from Riemannian geometry and from analysis, the theory of harmonic maps between Riemannian manifolds has developed in many diverse branches. In particular, there is now a whole battery of deep and interesting results about harmonic maps to or from complex manifolds and Kähler spaces.

Within almost contact geometry, there are several classes of manifolds that can be considered as odd-dimensional analogs of Kähler spaces, the most important ones being Sasakian and cosymplectic spaces.

The theory of harmonic maps on smooth manifolds endowed with some special structures has its origin in the paper of Lichnerowicz [6], in which he considered holomorphic maps between Kähler manifolds.

In general the construction of energy minimizing maps is much more difficult than finding harmonic ones. The main purpose of this paper is to show that structure-preserving maps on cosymplectic manifolds minimize the energy of maps. We prove that a (φ, J) -holomorphic map from a compact cosymplectic manifold to a Kähler manifold is not only harmonic but also a minimizer for its energy.

We also prove a converse of the previous result, that is, a smooth energy minimizer map from a cosymplectic manifold to a Kähler manifold, which is homotopic with a (φ, J) -holomorphic one is also (φ, J) -holomorphic.

2010 AMS *Mathematics subject classification*. Primary 53C25, 53C43, 58E20.

Keywords and phrases. Harmonic maps, cosymplectic manifolds, energy functional, holomorphic maps.

Partially supported by CEEX, 2-CEX 06-11-22/25.07.2006.

Received by the editors on May 2, 2008, and in revised form on June 14, 2008.

DOI:10.1216/RMJ-2010-40-6-1875 Copyright ©2010 Rocky Mountain Mathematics Consortium