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ASYMPTOTIC BEHAVIOR OF BOUNDED SOLUTIONS TO SOME SECOND ORDER EVOLUTION SYSTEMS

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ABSTRACT. By using previous results of Rouhani [**20–23**] for dissipative systems, we study the asymptotic behavior of solutions to the following system of second order evolution equation

$$\begin{cases} u''(t) - cu'(t) \in Au(t) & \text{a.e. } t \in (0, +\infty) \\ u(0) = u_0,\ \sup_{t \geq 0} |u(t)| < +\infty \end{cases}$$

where A is a maximal monotone operator in a real Hilbert space H and $c \geq 0$. We investigate weak and strong convergence theorems for solutions to this system. Our results extend and unify previous results by Mitidieri [**15**] and Morosanu [**17**] who studied the case $c = 0$ by assuming that A is maximal monotone and $A^{-1}(0) \neq \phi$, as well as previous results by Véron [**25**] who studied the case $c > 0$ by assuming A to be strongly monotone.

1. Introduction. Let H be a real Hilbert space with inner product $(.,.)$ and norm $|.|$. We denote weak convergence in H by w-lim and strong convergence by lim. $u'(t)$, respectively $u''(t)$, denotes $du/dt(t)$, respectively $d^2u/dt^2(t)$. A self-mapping T of a nonempty subset D of H is called nonexpansive, if: $|Tx - Ty| \leq |x - y|$ for all $x, y \in D$. Let A be a nonempty subset of $H \times H$ to which we shall refer as a (nonlinear) possibly multi-valued operator in H. A is called monotone, respectively strongly monotone, if $(y_2 - y_1, x_2 - x_1) \geq 0$, respectively $(y_2 - y_1, x_2 - x_1) \geq \alpha |x_1 - x_2|^2$ for some $\alpha > 0$, for all $[x_i, y_i] \in A$, $i = 1, 2$. A is maximal monotone if A is monotone and $R(I + A) = H$, where I is the identity operator on H.

Nonexpansive mappings as well as maximal monotone operators and semi-groups generated by them have been extensively studied. We

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