ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 40, Number 4, 2010

# MULTIPLIER HOPF ALGEBRAS IMBEDDED IN LOCALLY COMPACT QUANTUM GROUPS

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ABSTRACT. Let $(A, \Delta)$ be a locally compact quantum group and $(A_0, \Delta_0)$ a regular multiplier Hopf algebra. We show that if $(A_0, \Delta_0)$ can in some sense be imbedded in $(A, \Delta)$, then $A_0$ will inherit some of the analytic structure of $A$. Under certain conditions on the imbedding, we will be able to conclude that $(A_0, \Delta_0)$ is actually an algebraic quantum group with a full analytic structure. The techniques used to show this can be applied to obtain the analytic structure of a $^*$-algebraic quantum group *in a purely algebraic fashion*. Moreover, the *reason* that this analytic structure exists at all is that one-parameter groups, such as the modular group and the scaling group, are diagonalizable. In particular, we will show that necessarily the scaling constant $\mu$ of a $^*$-algebraic quantum group equals 1. This solves an open problem posed in [**13**].

**1. Introduction.** In [**20**], the second author introduced *multiplier Hopf algebras*, generalizing the notion of a Hopf algebra to the case where the underlying algebra is not necessarily unital. In [**21**], he considered those multiplier Hopf algebras that have a nonzero left invariant functional. It turned out that these objects, termed *algebraic quantum groups*, possess a rich structure, allowing for example a duality theory. These objects seemed to form an algebraic model of locally compact quantum groups, which at the time had no generally accepted definition.

In [**13**], Kustermans showed that a $^*$-algebraic quantum group (which is an algebraic quantum group with a well-behaving $^*$-structure) naturally gives rise to a $C^*$*-algebraic quantum group*, which was a proposed definition for a locally compact quantum group by Masuda, Nakagami and Woronowicz [**16**]. Kustermans showed, however, that there was one discrepancy with the proposed definition, in that the invariance of the scaling group with respect to the left Haar weight was only relative.

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Received by the editors on June 5, 2007, and in revised form on February 19, 2008.
DOI:10.1216/RMJ-2010-40-4-1149 