ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 40, Number 4, 2010

QUASIANALYTIC GELFAND-SHILOV SPACES WITH APPLICATION TO LOCALIZATION OPERATORS

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ABSTRACT. We study localization operators with symbols in spaces of quasi-analytic distributions. More precisely, it is shown that certain quasianalytic distributions, considered as symbols, give rise to trace-class localization operators. We give a new structure theorem for quasianalytic distributions which combines its local and global properties. In the proof we use the heat kernel and parametrix techniques, while in the study of localization operators we use the techniques of time-frequency analysis.

1. Introduction. Localization operators (anti-wick operators, Toeplitz operators, Gabor multipliers) are pseudo-differential operators $A_a^{\varphi_1,\varphi_2}$, where a is the symbol of the operator and φ_1, φ_2 are the analysis and synthesis windows, respectively (see below for an explicit expression). With respect to the classical pseudodifferential calculus, one may consider singular symbols for localization operators and nevertheless obtain good properties, in particular L^2-boundedness, see [**6**–**8**, **25**, **26**]. As an example, it was observed in [**8**] that certain compactly supported ultra-distributions give rise to trace-class operators. In this paper we study localization operators in the framework of quasianalytic distributions.

The support of a quasianalytic distributions cannot be defined. Therefore, in order to give a reasonable generalization of the results from [**6**, **8**, **26**], we present a technique which may, in a certain sense, describe the local behavior of a quasianalytic distribution. More precisely, we give a new representation theorem for the class of quasi-

2010 AMS *Mathematics subject classification.* Primary 47G30, 35S05, 46F05, 44A05.
Keywords and phrases. Localization operator, ultra-distributions, Gelfand-Shilov spaces, modulation space, Wigner distribution, short-time Fourier transform, Schatten class.
This research is supported by FIRB (MIUR, Italy), MNTR (Serbia) through project 144016 and UNESCO BRESCE through AFC 875.871.6.
Received by the editors on August 2, 2007, and in revised form on March 13, 2008.
DOI:10.1216/RMJ-2010-40-4-1123