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NUMBER-THEORETIC CONDITIONS WHICH YIELD ISOMORPHISMS AND EQUIVALENCES BETWEEN MATRIX RINGS OVER LEAVITT ALGEBRAS

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ABSTRACT. For each integer $n \geq 2$ let L_n denote the Leavitt algebra of order n. We provide number-theoretic descriptions of the relationships between the integers k, k', n, n' for which there are isomorphisms and/or equivalences between the matrix rings $\mathrm{M}_k(L_n)$ and $\mathrm{M}_{k'}(L_{n'})$ possessing various properties. Such properties include: isomorphism (unrestricted), induced isomorphism, graded isomorphism and graded equivalence. These results extend the isomorphism results achieved in [**2**].

Throughout this note K denotes a field. For $n \geq 2$ we denote by $L_K(1,n)$, or simply L_n when appropriate, the *Leavitt algebra of order n with coefficients in K*. $L_K(1,n)$ is the free associative K-algebra with generators $\{x_i, y_i : 1 \leq i \leq n\}$ and relations

$$x_i y_j = \delta_{ij} \text{ for all } 1 \leq i, j \leq n, \quad \text{and} \quad \sum_{i=1}^{n} y_i x_i = 1.$$

(See [**2**] or [**10**] for additional information about L_n.) $R = L_n$ also may be viewed as the K-algebra universal with respect to the property that ${}_RR \cong {}_RR^n$ as left R-modules. Indeed, an important explicit isomorphism $\phi : {}_RR \to {}_RR^n$ is given by

$$\phi(r) = (ry_1, ry_2, \ldots, ry_n), \text{ with inverse } \phi^{-1}((r_1, r_2, \ldots, r_n)) = \sum_{i=1}^{n} r_i x_i$$

for all $r \in R$ and $(r_1, r_2, \ldots, r_n) \in R^n$.

There has been recent sustained interest in Leavitt algebras, for two important reasons. First, connections between the Leavitt algebras and their C*-algebra counterparts, the so-called *Cuntz algebras* $\mathcal{O}_n$,

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