ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 40, Number 3, 2010

EXISTENCE OF NONSTATIONARY PERIODIC SOLUTIONS OF Γ-SYMMETRIC ASYMPTOTICALLY LINEAR AUTONOMOUS NEWTONIAN SYSTEMS WITH DEGENERACY

JUSTYNA FURA, ANNA GOŁĘBIEWSKA AND HAIBO RUAN

ABSTRACT. For a finite group Γ, we consider a Γ symmetric autonomous Newtonian system, which is asymptotically linear at ∞ and has 0 and ∞ as isolated degenerate critical points of the corresponding energy function. By means of the equivariant degree theory for gradient G-maps with $G = \Gamma \times S^1$, we associate to the system a topological invariant $\deg_\infty - \deg_0$, which is computable up to an unknown factor due to the degeneracy of the system. Under certain assumptions, this invariant still contains enough information about the symmetric structure of the set of periodic solutions, including the existence, multiplicity and symmetric classification. Numerical examples are provided for Γ being the dihedral groups D_6, D_8, D_{10}, D_{12}.

1. Introduction. Consider a finite group Γ, which is a symmetry group of certain regular polygon or polyhedron in $\mathbf{R}^n$, and define a Γ-action on $V := \mathbf{R}^n$ by permuting the coordinates of the vectors $x \in V$. In particular, V is an *orthogonal* Γ-representation with respect to the usual Euclidean metric. The goal of this paper is to study, in the presence of Γ-*symmetry*, the existence of nonstationary periodic solutions $x : \mathbf{R} \to V$ of the following autonomous Newtonian system

$$(1.1) \qquad \begin{cases} \ddot{x} = -\nabla\varphi(x), \\ x(0) = x(2\pi), \quad \dot{x}(0) = \dot{x}(2\pi), \end{cases}$$

where $\varphi : V \to \mathbf{R}$ is a C^2-differentiable Γ-invariant function such that $(\nabla\varphi)^{-1}(0) = \{0\}$ and $\nabla\varphi$ is asymptotically linear at infinity, i.e., there

Research of the first author partially sponsored by the Doctoral Program in Mathematics at the Nicolaus Copernicus University, Toruń, Poland and by the Ministry of Education and Science, Poland, under grant N N201 385534. Research of the second author partially supported by the Ministry of Education and Science, Poland, under grant N N201 385534 and by Nicolaus Copernicus University, Toruń, Poland, under grant 384-M. Research of the third author supported by Izaak Walton Killam Memorial Scholarship, University of Alberta, Canada.
Received by the editors on October 19, 2007, and in revised form on February 10, 2008.

DOI:10.1216/RMJ-2010-40-3-873