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RINGS OVER WHICH ALL MODULES ARE STRONGLY GORENSTEIN PROJECTIVE

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ABSTRACT. One of the main results of this paper is the characterization of the rings over which all modules are strongly Gorenstein projective. We show that these kinds of rings are very particular cases of the well known quasi-Frobenius rings. We give examples of rings over which all modules are Gorenstein projective but not necessarily strongly Gorenstein projective.

1. Introduction. Throughout this paper all rings are commutative with identity element and all modules are unital. It is convenient to use "m-local" to refer to (not necessarily Noetherian) rings with a unique maximal ideal m.

For background on the following definitions, we refer the reader to [**3, 5–7**].

Definition 1. A module M is said to be *Gorenstein projective* if there exists an exact sequence of projective modules

$$\mathbf{P} = \cdots \longrightarrow P_1 \longrightarrow P_0 \longrightarrow P^0 \longrightarrow P^1 \longrightarrow \cdots$$

such that $M \cong \operatorname{Im}(P_0 \to P^0)$ and such that $\operatorname{Hom}(-, Q)$ leaves the sequence $\mathbf{P}$ exact whenever Q is a projective module.

The exact sequence $\mathbf{P}$ is called a *complete projective* resolution.

The *Gorenstein injective* modules are defined dually.

Recently in [**3**], the authors studied a simple particular case of Gorenstein projective and injective modules, which are defined, respectively, as follows:

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