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# ON THUE EQUATIONS OF SPLITTING TYPE OVER FUNCTION FIELDS

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ABSTRACT. In this paper we consider Thue equations of splitting type over the ring $k[T]$, i.e., they have the form

$$X(X-p_1Y)\cdots(X-p_{d-1}Y)-Y^d=\xi,$$

with $p_1,\dots,p_{d-1}\in k[T]$ and $\xi\in k$. In particular, we show that such Thue equations have only trivial solutions provided the degree of $p_{d-1}$ is large, with respect to the degree of the other parameters $p_1,\dots,p_{d-2}$.

**1. Introduction.** Let $F\in \mathbf{Z}[X,Y]$ be a homogeneous, irreducible polynomial of degree $d\geq 3$. Then the Diophantine equation

$$F(X,Y)=m,\quad m\in\mathbf{Z}\setminus\{0\}$$

is called a Thue equation in honor of Axel Thue [**23**] who proved the finiteness of the number of solutions. Since then several Thue equations and also families of Thue equations were solved. In particular, families of Thue equations of the form

$$X(X-a_1Y)\cdots(X-a_{d-1}Y)+Y^d=\pm 1, \tag{1}$$

with $a_1,\dots,a_{d-1}$ were studied by several authors, e.g., Heuberger [**9**], Lee [**12**], Mignotte and Tzanakis [**16**], Pethő [**18**], Pethő and Tichy [**19**], Thomas [**22**] and Wakabayshi [**24**]. This type of Thue equation is called splitting type. Obviously these Thue equations have solutions $\pm(1,0),\pm(0,1),\pm(a_1,1),\dots,\pm(a_{d-1},1)$, which are called trivial. Thomas [**22**] investigated Thue equations of splitting type of degree $d=3$ with $a_1=p_1(n)$, $a_2=p_2(n)$, where $p_1,p_2$ are monic polynomials

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