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RANDIĆ INDEX AND EIGENVALUES OF GRAPHS

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ABSTRACT. Let G be a simple connected graph. The general Randić index $R_\alpha(G)$ of G is defined as $R_\alpha(G) = \Sigma_{uv\in E(G)}(d(u)\,d(v))^\alpha$. In this paper, we present upper and lower bounds for $R_{-1}(G)$ in terms of the normalized Laplacian eigenvalues of a graph.

1. Introduction. In this paper, we use the standard notation in graph theory in [**4**]. Let $G = (V(G), E(G))$ be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$, $|V(G)| = n$ and $|E(G)| = m$. The general Randić index $R_\alpha(G)$ of G is defined as the sum of $(d_u d_v)^\alpha$ over all edges uv of G, where d_u denotes the degree of $u \in V(G)$, i.e.,

$$R_\alpha(G) = \Sigma_{uv\in E(G)}(d_u d_v)^\alpha,$$

where α is an arbitrary real number.

It is well known that $R_{-1/2}$ was introduced by Randić in 1975 as one of the many graph theoretical parameters derived from the graph underlying some molecule [**13**]. Like other successful chemical indices, this has been closely correlated with many chemical properties. The general Randić index was proposed by Bollobás and Erdös [**3**], and Amic et al. [**1**], independently, in 1998. It has been extensively studied by both mathematicians and theoretical chemists. Many important mathematical properties have been established. Li and Yang [**11**] studied the general Randić index for general graphs and obtained lower and upper bounds for the general Randić index among graphs of order n. Araujo et al. [**2**], Lu et al. [**12**], Gutman et al. [**6, 7**] and Xiao et al. [**14**] studied the general Randić index in terms of eigenvalues of the Laplacian matrix and the adjacent matrix of a graph. Later Hu

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