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THE THREE BODY PROBLEM WITH A RIGID BODY: EULERIAN EQUILIBRIA AND STABILITY

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ABSTRACT. We consider the noncanonical Hamiltonian dynamics of a rigid body in the three body problem. By means of geometric-mechanics methods we will study the approximate dynamics that arise when we develop the potential in a series of Legendre and truncate the series to the second harmonics. Working in the reduced problem, we will study the existence of equilibria that will dominate Euler in analogy with classic results on the topic. In this way, we generalize the classical results on equilibria of the three-body problem and many of those obtained by other authors using more classic techniques for the case of rigid bodies. The instability of Eulerian equilibria is proven in this approximate dynamics if the rigid body is close to the sphere.

1. Introduction. In the study of configurations of relative equilibria by differential geometry methods or by more classical ones we will mention here the papers of Wang et al. [**8**] in regards to the problem of a rigid body in a central Newtonian field and Maciejewski [**3**] in regards to the problem of two rigid bodies in mutual Newtonian attraction.

For the problem of three rigid bodies we would like to mention that Vidyakin [**7**] and Dubochine [**1**] proved the existence of Euler and Lagrange configurations of equilibria when the bodies possess symmetries; Zhuravlev and Petrutskii [**10**] made a review of the results up to 1990. These works use canonical variables for the deduction of their results.

In Vera [**4**] and a recent paper of Vera and Vigueras [**6**] we study the noncanonical Hamiltonian dynamics of $n+1$ bodies in Newtonian attraction, where n of them are rigid bodies with spherical distribution

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